

Syllabus of the course for PhD students of Mathematics within the Doctoral School of Exact and Natural Sciences

Title: Sparse equidistribution problems in dynamics

Coordinator: Adam Kanigowski

Programme: mathematics

Number of hours and form of classes: 30 h lectures

Scheduled date of classes: spring semester 2024/2025

Form of crediting the course: For passing grade (ZAL) attendance is requested. For the exam (scale 3.0, 3.5, ..., 5.0) there will be an oral discussion.

Prerequisites: Some basic analysis, measure theory and basic number theory. Some knowledge of ergodic theory should be helpful.

MSc/BSc students: yes, MSc students are welcome.

Course topics (in short):

Let (X, T) be a topological dynamical system. Classical ergodic theory studies asymptotic behavior of averages of the form $\frac{1}{N} \sum_{n \leq N} f(T^n x)$ where $x \in X$ and $f \in C(X)$. There are many fundamental results on convergence of such averages in various norms: the supremum norm, almost everywhere and in L^2 being the main examples (in the latter two we equip (X, T) with a measure μ which is preserved by T). Recently there has been a lot of interest in studying non-classical averages. In the most general form, given a bounded sequence (a_n) one is interested in the asymptotic behavior of averages:

$$\frac{1}{\sum_{n \leq N} |a_n|} \sum_{n \leq N} a_n \cdot f(T^n x). \quad (1)$$

In most applications the sequence (a_n) is a structured sequence coming from number theory. Some of the examples of such sequences include:

1. a_n = Möbius function or Liouville function (or more generally a multiplicative function).
2. for $k \geq 1$, $a_n = 1$ if n is a product of at most k primes, $a_n = 0$ otherwise.
3. $a_n = 1$ if n is a k -th power (like square) and $a_n = 0$ otherwise.
4. (a_n) a random sequence of real numbers.

The goal of the course is to give an overview and recent advancement in studying (1). The nature of the problem usually makes it interesting both from the dynamical and number theory perspective.

Literature:

1. *Ergodic Theory*, Cornfeld, Fomin, Sinai.
2. *Pointwise ergodic theorems via harmonic analysis*, Rosenblatt, Wierdl.